FRACTAL CHARACTERISTICS OF WORLD MARKET COMMODITY DERIVATIVES

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ABSTRACT: *The fractal properties of some segments of the global commodity derivatives market have been investigated in the present article. The fractal nature of segments of oil and copper derivatives markets has been determined. The presence of speculative reference groups of investors on given segments has been substantiated.*

Keywords: derivatives, clusters of investors, Hurst exponent, fractal analysis, fractal dimension.

JEL Codes: F30

Introduction and problem statement.

The processes taking place in the global derivatives market are of special interest to scientists. This is due to the fact that the derivatives market is the largest segment of the global financial market , which is largely self-sufficient market, where profits are obtained simply by investing money in derivatives and their subsequent sale , etc., i.e. as a result of financial speculation (Радаєва М,2010.),(Rubtsov B.B.,2011), (Sokhats'ka O.,2010), (Shapovalova O.,2012). However, self-sufficiency of the derivatives market does not mean the absence of its impact on other segments of the financial system and world economy on the whole. The 2008 events are evidence of this destructive influence, which, as scientists rightfully claim, were caused by certain segments of derivatives market. Therefore, the study of characteristic processes in the derivatives market is of particular importance.

The problem is that today financial markets tend to acquire systemic characteristics, they transform into complete systems capable of self-organization, evolution in different modes, some of which may end with crises of various sizes, after which the market goes into a different mode. One manifestation of self-organization is fractality, which can be found in nature. Its essence can be interpreted in what follows. If there is a system with reverse nonlinear feedback, the input characteristics will be related to the output ones, but there will always be some uncertainty fraction. Past market behavior is input characteristic for the behavior of financial markets, the output characteristics form the projected behavior (Dubovnikov M.M.,2011).

If the market is characterized by fractality, the efficient market hypothesis will not be true for it, the dynamics of processes will be determined by coherent actions of one or more reference investors groups who can operate synchronously or asynchronously[2]. If fractality is absent, market behavior is described by "random wandering", which presupposes stochastic (random) changes in the market and leads, in turn, to a normal distribution of prices / profitabilities.

Various scholars have repeatedly emphasized that the derivatives market is speculative in nature (Sokhats'ka O., 2010), (Shapovalova O.,2012), (Shevelev B.). If this argument is valid, there should be at least one big reference group of investors (financial speculators) in the derivatives market who coordinate their activity. Hence, the derivatives market should be fractal.

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[2] Reference group in this case refers to a group of investors that operate by the same rules

The aim of the present article is to evaluate fractal characteristics of certain segments of the global derivatives market characterized by presence of long numerical series of historical data to split them into before crisis, pre-crisis and post-crisis periods.

Review of latest publications.

Sufficiently profound generalization of fractal analysis capabilities of numerical series investigation that characterize financial markets was conducted in the works of E. Peters, who defined fractal characteristics of the shares market on the basis of the Hurst index and laid the methodological foundations of the application of the so-called FDI, or R/S-analysis to the study of financial markets (Peters E., 2004), (Peters E.,2000).

In the works of N. Maksyshko, fractality of prices in the world gold market is determined (Maksyshko N.K.,2008). However, the author, having determined the fractality availability in the abovementioned market, limits himself to conclusions about inapplicability of traditional econometric methods to its forecasting. In addition, spot market was investigated. Fractality of spot price of gold was also determined by O. Mykhailovska (Mykhailovka O.V., 2011). Both authors prove inapplicability of the efficient market theory to the gold market and all models developed around it. However, gold is a specific commodity, which can be regarded both as a raw material and as a financial asset (Mykhailovka O.V., 2011). Thus, the problem whether commodity derivatives markets are fractal by their essence, in line with other financial markets, has not yet been profoundly investigated in the scientific literature.

The main part.

To study the fractal properties of the global commodity derivatives markets, we use the R/S analysis, based on the works of E. Peters (Peters E., 2004), (Peters E.,2000). Algorithm of actions by the method of this author can be presented in the simplified form:

1) calculation of the Hurst exponent H;

2) determination of the fractal dimension index. $D = 2 - H$;

3) interpretation of results.

Two ways are used to calculate the Hurst exponent. The first one involves the following calculation steps (Peters E., 2004), (Peters E.,2000) :

1. M_t - average values ΔR_t for time intervals t = 0, 1, 2, ..., n – 2 are calculated

$$\grave{I}_t = \frac{\sum_{i=1}^{t} \Delta R_i}{t+1}.$$ (1)

2. Accumulated deviations $\tilde{O}_t$ for each interval t are calculated

$$\tilde{O}_t = \sum_{i=0}^{t} \left(\Delta R_i - M_i \right).$$ (2)

3. Maximal ($\max_t(X_1, ...X_t)$) and minimal ($\min_t(X_1, ...X_t)$) deviation over $n – 1$ periods as well as scale of accumulated deviation R_t under various t.

$$R_t = \max_t(X_1,..X_t) - \min_t(X_1,..X_t).$$ (3)

4. Root-mean-square deviation S_t for each interval is calculated

$$S_t = \sqrt{\frac{\sum_{i=0}^{t} \left(\Delta R - M_t \right)^2}{t+1}}.$$ (4)

5. Normalization of accumulated deviation scale S is performed (a series of R / S values is obtained for each interval t).

6. R/S and *t* are taken the logarithm of, subordination graph of $\log\left(\dfrac{R}{S}\right)$ against $\log(t)$ is projected.

7. The method of least squares is used to determine linear approximation (value of the angular coefficient of linear regression is a measure of Hurst index), then the fractal dimension D = 2 – H. is calculated.

1. The above illustrated algorithm is called a fast algorithm. There is a "slow" algorithm, which includes all the above mentioned steps, but for each time period *t* shift along the series is performed (Gamaley Ya.V., 2008).

The series is divided into several ranges of length *t*, for each of which the rate of R/S is determined by, then the mean value is calculated and positioned on the graph to determine the Hurst exponent. However, the second method gives very general information throughout the series and should be used with particular caution in research of investment process, whose fractal characteristics may change significantly. Therefore, we will use the first method.

Depending on fractal dimensions, there are different modes of financial markets operation.

1. In case when fractal dimension is $D < 1.4$ (black noise), the market is developing in one direction under the influence of some "organizing force" (market is affected by some external influence or existing dominant one reference group of investors).

2. If fractal dimension is $1.4 < D < 1.6$ (brown noise), the market is developing in stochastic mode, in line with the efficient market hypothesis.

3. The range of values $D > 1.6$ (pink noise) corresponds to the mode when market is evolving under the influence of several "organizing forces" acting in one direction (such as the effect of certain external factors influencing synchronous actions of members of the reference group of investors). In this mode, the market develops unstable condition, which can be replaced with the new order of organization (high probability of trend different from what is observed in the current period).

4. The instance of $D < 1$ corresponds to the opposite action of the "organizing forces", such mode is characterized by a high probability of abrupt change of direction (chaotic changes occur more frequently than in the stochastic process).

It should be noted that the Hurst index and fractal dimension calculated on the basis of it are integral of a process that reflects the analyzed series. But the modes in which financial markets evolve may vary through different time periods. Derivatives markets are no exception. To illustrate this, Figure 1 shows dynamics of oil prices on futures contracts.

Visual analysis of the Fig. 1 enables differentiation of three time ranges with different market operation modes.

The first period of 1990-2000 is characterized by minor fluctuations in prices at a relatively slow average level change. The second period of 2001-2008 is characterized by increased fluctuations with significant non-linear changes in the equilibrium level, which led to a significant fall in prices (almost thrice). The third period of 2009-2013 is characterized by the same non-linear average level changes and even greater scale fluctuations. Therefore, the fractal analysis is logical to be conducted at each time interval, where there is reason to believe, that there was a change in market operation mode.
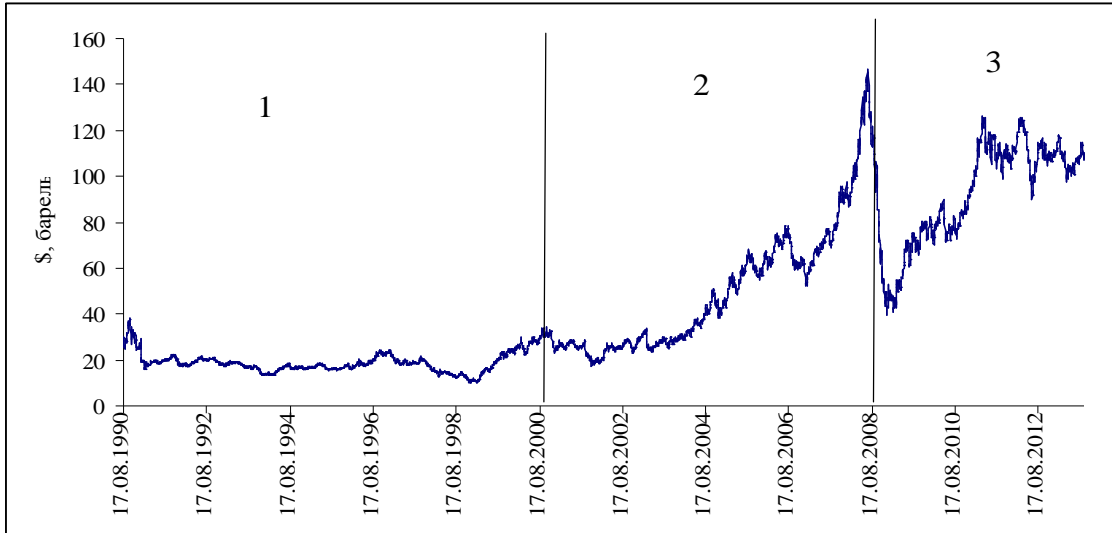
Figure no. 1. - Dynamics of prices for Brent crude oil on futures contracts on the Intercontinental Exchange (Europe)
Source: elaborated by the author according to (ICE Brent Crude Oil Future).

The estimated values of the Hurst exponent using Fractan 4.4 software (by V. Sychev) are illustrated in table. 1.

Table no.1

Hurst index and fractal dimension at different time intervals in the futures market for Brent oil

№	Time period	Hurst exponent	Fractal dimension
1	1990-2013	$H = 1.082 \pm 0.136$	$D = 0.918 \pm 0.136$
2	1990-2000	$H = 0.979 \pm 0.102$	$D = 1.021 \pm 0.102$
3	2001-2008	$H = 0.960 \pm 0.051$	$D = 1.040 \pm 0.051$
4	2009-2013	$H = 0.982 \pm 0.229$	$D = 1.018 \pm 0.229$

Source: calculated by the author according to (ICE Brent Crude Oil Futures).

Analyzing data calculations presented in Table 1 we can draw the following conclusions.

First, fractal dimension of the processes taking place in the oil futures market significantly differs from the values range $D \in [1.4; 1.6]$. This means that these processes are not stochastic. The hypothesis of market efficiency and all "classic" models build around it do not hold for the oil futures market.

Second, at all time periods of $H > 0.5$, i.e. processes in oil futures markets are characteristic of "black noise". Moreover, at all the examined time periods, Hurst exponent acquires enough close values, all trusting intervals overlap. However, at all time periods, it is significantly higher than 0.5. and approaches quite close to the range of $H > 1$, which corresponds to a "disaster noise", illustrated by the events in the autumn of 2008, and which might be called "market crash" for oil futures. However, these developments have not led to significant changes in the fractal characteristics of the market, it returned to the mode of "black noise" after 2009.

Third, the value of the Hurst index and fractal dimension lead to a conclusion that, overall, commodity futures market develops under the influence of sufficiently significant reference group of investors. However, it does provide any possibility to answer the question: Why is dynamics of oil prices futures different on three ranges?

2. To answer the above question, we need to turn to another possibility offered by the R / S analysis to establish market characteristics. We are talking in this case about a R/S time subordination graph in logarithmic coordinates. As substantiated by E. Peters, "failure", or abrupt downward deflection of the graph indicate the time investment horizon of the reference group of investors, if available (Peters E. 2004., Peters E. 2000.)

The fig. 2 illustrates the corresponding graphs for all four time ranges, according to table 1.
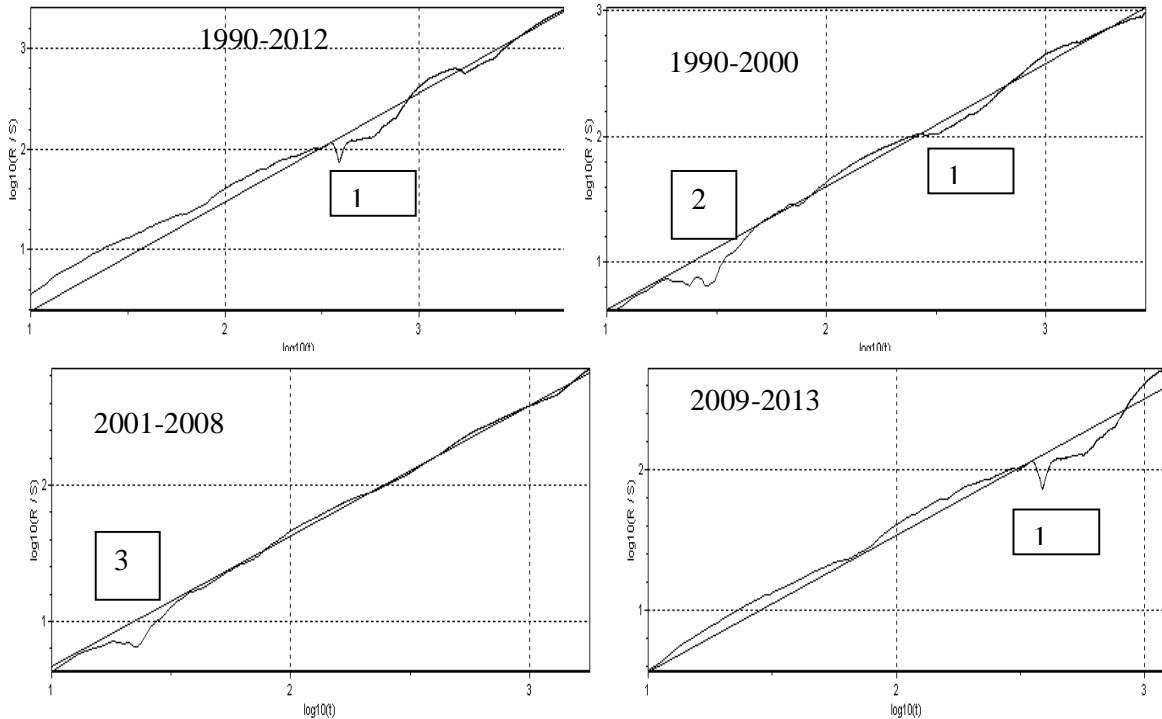

Figure no. 2. - Graphs of R/S time subordination in logarithmic coordinates for oil futures
*Source: calculated by the author according to (*ICE Brent Crude Oil Futures*).*

Analyzing the graphs shown in Figure 2, we can draw the following conclusions. In the time period of 1990-2013, 1990-2000, 2009-2013, the investment horizon of the reference group of investors (Figure 2 indicates graph deviation of "1") was $T_1 = 10^{2.5} \approx 316\,\text{days}$, or almost a year.

In the period of 2002-2008, there was a significant increase in that segment of the second reference group of investors with a time horizon of (Fig. 2 corresponds to the deviation "2") $T_2 = 10^{1.4} = 25$ days.

It is logical to assume that risk capital began to flow to the oil derivatives market in this period, and as a result the second reference group of investors was formed in the market with a time horizon of about 25 days. Its increase occured in the next time period of 2001-2008, wherein the referred group became dominant in the market, as the graph deviation corresponding to time $T_1 = 316$ days is not visible on that graph. Thus, the oil derivatives market of 2001-2008 became truly speculative in nature, with a typical investment horizon of operations dropping to $T_3 = 15 - 20$ days.

Drawing parallels with the behavior of the price of oil futures (Fig. 1), we can conclude that the continued dominance of the reference group of speculative investors in the market caused significant price increases.

"Market crash" in autumn 2008, which was an end to the long process of increasing prices of oil futures manifested, itself in the retreat of a large share of investors from the market with short

investment horizon. In the period of 2009-2013, again there is a deviation, which corresponds to the investment horizon of $T_1 = 316$ days.

So, the same reference group of investors as in the period of 1990-2000 was dominant. It can be assumed that there was no total outflow of speculative capital from the oil futures market after 2008, as indicated by a high range of market fluctuations (fig. 1). Moreover, experts point to the increasing inflow of speculative capital into the commodity derivatives market at the end of 2009 (Sokhats'ka O., 2010).

However, the R / S analysis does not give an opportunity to conclude that this group of investors is visible in the market in the period of 2009-2013. However, this does not mean that the oil futures market will not become speculative in the future.

Let us analyze a different segment of the global commodity derivatives market, which a segment of the so-called "copper" futures. It should be noted that the price situation in this segment resembles the dynamics of oil futures prices.
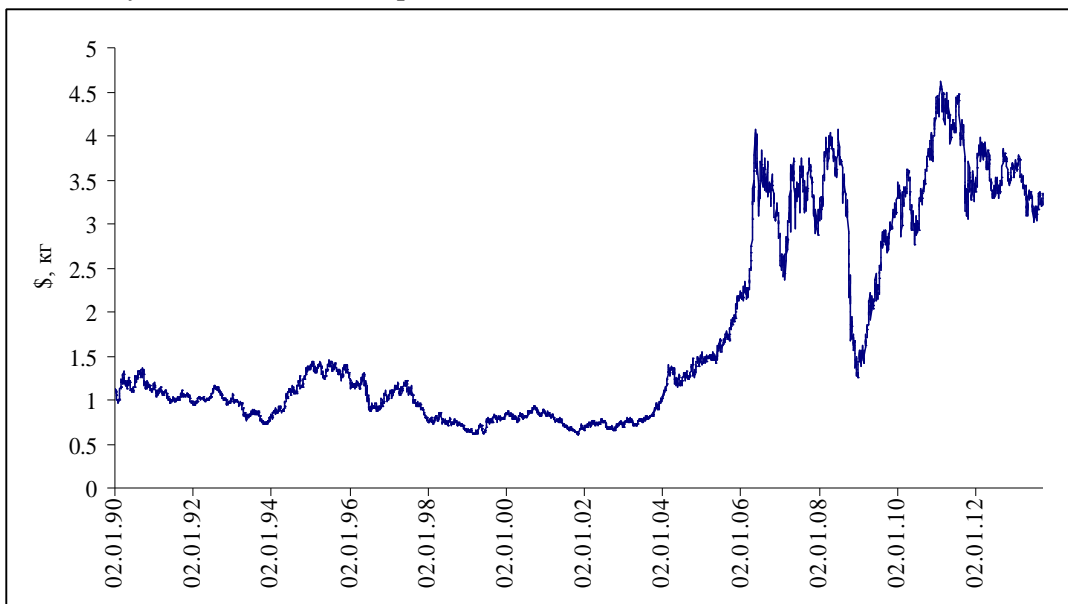

***Figure no.3. -* Prices dynamics for copper futures**
*Source: elaborated by the author according to (*ICE Brent Crude Oil Futures*).*

The figure 3 illustrates the "crash" in the late 2008, which was followed by a significant increase in prices.

Table no. 2.

**Hurst exponent and fractal dimension at different time intervals
in the futures market for copper**

№	Time period	Hurst exponent	Fractal dimension
1	1990-2013	$H = 1.078 \pm 0.152$	$D = 0.922 \pm 0.152$
2	1990-2000	$H = 0.995 \pm 0.090$	$D = 1.005 \pm 0.090$
3	2001-2008	$H = 1.047 \pm 0.156$	$D = 0.953 \pm 0.156$
4	2009-2013	$H = 0.954 \pm 0.112$	$D = 1.046 \pm 0.112$

*Source: calculated by the author according to (*ICE Brent Crude Oil Futures*).*

As in the case of oil derivatives market dynamics, we can conclude that, regardless of the time period, copper futures market is also fractal, hence Hurst exponent is significantly different from 0.5 for all intervals. The specified segment is similar to the segment of petroleum derivatives

and that the market is well organized under one "organizing force" as the Hurst index exceeds 0.5. However, the deviation of R / S graph from *t* in logarithmic coordinates has significant differences from similar deviations for oil (Fig. 4).
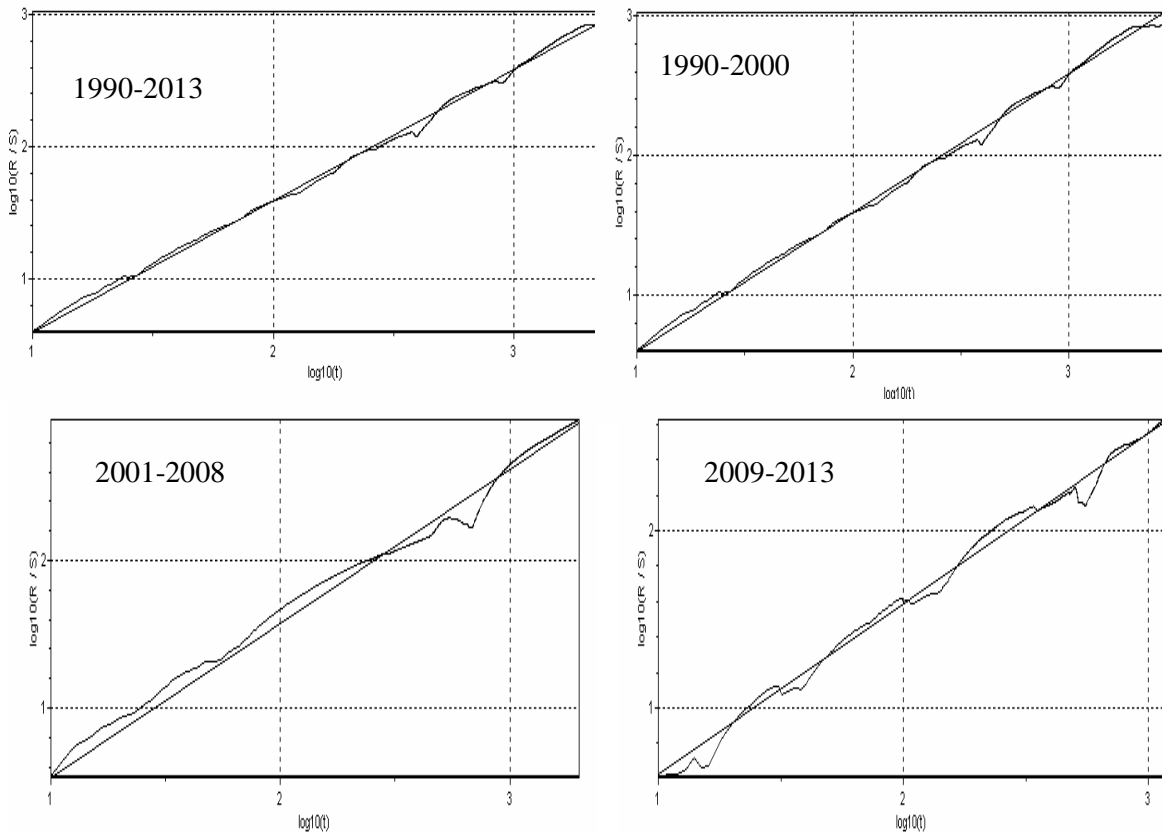

Figure no. 4. - Graphs of R/S time subordination in logarithmic coordinates for copper futures
*Source: calculated and elaborated by the author according to (*ICE Brent Crude Oil Futures) .

The difference is that there is no deviation on the interval of 1990-2000, which would indicate dominance of reference groups of investors with short investment time horizons. This correlates with the opinion of experts, which indicate the presence of major "players" in the "copper" derivatives market in form of big corporations that also play a speculative game, but with a fairly large-scale time investment horizon (all time periods are characterized with a deviation, which correspond, as in the case of oil derivatives, to the time span of approximately 300-350 days). At the same time, emergence of a significant group of investors with short-term time horizons in the post crisis period of 2009-2013 should be taken into account. Therefore, we can conclude that the "crashes" in the market may be caused by the operations of investors with short-term investment horizons as well as by investors with long-term time horizons.

Conclusions and prospects for further research.
Thus, in the case of futures on crude oil and copper, it can be concluded that market processes in these segments are fractal according to the "black noise" type. Furthermore, Hurst index has a value, which is slightly different from one. This indicates that on certain local segments market processes can quickly adjust in such way that $H > 1$ will be performed in a certain time period , which means high probability of market crises. The reason for this mode of commodity derivatives market dynamics is presence of a large number of speculative investors, who can reduce or increase the investment time horizons. It should be noted that crisis can be caused by both

investors with short-term investment horizons (oil derivatives) and by investors with long-term time horizons (copper derivatives).

Fractal of the commodity derivatives market segments under study means that the classical methods of market risk, profits and behavior forecasting are inapplicable to them, as all market processes are not stochastic. Search for new models, which could account for fractal nature of the processes in commodity derivatives markets. Presence and even prevalence of speculative investors on these segments indicates a need for stronger regulation of commodity derivatives transactions for the reason that crises in these markets have a direct impact on the crises in real sectors of world economy, as they affect the interests of primary sector producers and its consumers.

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